International Workshop

« Complex Affine Geometry, Hyperbolicity, Complex Analysis »

Speakers and Abstracts

Ivan V. Arzhantsev (HSE, Moscou)
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Title : Additive Actions on Toric Varieties

Abstract : By an additive action on an algebraic variety X of dimension n we mean a regular action $\mathbb{G}_a^n \times X \to X$ with an open orbit of the commutative unipotent group $\mathbb{G}_a^n$. We begin with a survey of results on additive actions including Hassett-Tschinkel's correspondence for projective spaces and the case of (generalized) flag varieties.
We prove that if a complete toric variety X admits an additive action, then it admits an additive action normalized by the acting torus. Normalized additive actions on a toric variety X are in bijection with complete collections of Demazure roots of the fan of X. Moreover, any two normalized additive actions on X are isomorphic.
This is a joint work with Elena Romaskevich.

Fabrizio Catanese (U. Bayreuth)
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Title : New examples of rigid varieties

Abstract : Given an algebraic variety defined by a set of equations, an upper bound for its dimension at one point is given by the dimension of the Zariski tangent space.
The infinitesimal deformations of a variety X play a somehow similar role, they yield the Zariski tangent space at the local moduli space, when this exists, hence one gets an efficient way to estimate the dimension of a moduli space.
It may happen that this moduli space consists of a point, or even a reduced point if there are no infinitesimal deformations. In this case one says that X is rigid, respectively inifinitesimally rigid.
A basic example is projective space, which is the only example in dimension 1.
In the case of surfaces, infinitesimally rigid surfaces are either the Del Pezzo surfaces of degree ≥ 5, or are some minimal surfaces of general type.
As of now, the known surfaces of the second type are all projective classifying spaces (their universal cover is contractible), and have universal cover which is either the ball or the bidisk (these are the noncompact duals of $\mathbb{P}^2$ and $\mathbb{P}^1 \times \mathbb{P}^1$),

or are the examples of Mostow and Siu, or the Kodaira fibrations of Catanese-Rollenske.
Together with ingrid Bauer, we showed the rigidity of a class of surfaces which includes the BCD surfaces and the Hirzebruch-Kummer coverings of the plane branched over a complete quadrangle.
I will finish mentioning other examples and several interesting open questions.

Ivan Cheltsov (U. Edinburgh)
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Title : Cylinders in del Pezzo surfaces

Abstract : For an ample divisor H on a variety V, an H-polar cylinder in V is an open ruled affine subset whose complement is a support of an effective Q-divisor that is Q-rationally equivalent to H. In the case when V is a Fano variety and H is its anticanonical divisor, this notion links together affine, birational and Kahler geometries. In my talk I will show how to prove existence and non-existence of H-polar cylinders in smooth and mildly singular del Pezzo surfaces for different ample divisors H. As an application, I will answer an old question of Zaidenberg and Flenner about additive group actions on the cubic Fermat affine threefold cone. This is a joint work with Park and Won.

Ciro Ciliberto (U. Roma Tor Vergata)
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Title : Infinitesimal Newton–Okounkov bodies

Abstract : Given a smooth projective algebraic surface X, a point $O \in X$ and a big divisor D on X, one can consider the set of all Newton–Okounkov bodies of D with respect to valuations of the field of rational functions of X centred at O, or, equivalently, with respect to a flag (E, p) which is *infinitely near* to O : this means that there is a sequence of blowups $X' \to X$, mapping the smooth, irreducible rational curve $E \subset X'$ to O. The objective of this talk is to present some experimental work on the study of such *infinitesimal* Newton–Okounkov bodies, and specifically on their variation as (E, p) varies. I will focus on the case $X = \mathbb{P}^2$.
This is joint work with Michal Farnik, Alex Küronya, Victor Lozovanu, Joaquim Roé and Constantin Shramov.

Thai Do Duc (Hanoi National Educational U.)
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Title : On integral points off divisors in subgeneral position in projective algebraic varieties

Abstract : The purpose of this talk is to present the following in Diophantine Geometry.
1. The first is to show the dimension of the set of integral points off divisors in subgeneral position in a projective algebraic variety $V \subset \mathbb{P}^m_{\overline{k}}$, where k is a number

field. As its consequences, the results of Ru-Wong [RW], Ru [R93], Noguchi-Winkelmann [NW], Levin [Le] are recovered.
2. The second is to show the complete hyperbolicity in the sense of Kobayashi of the complement of divisors in subgeneral position in a projective algebraic variety $V \subset \mathbb{P}^m_{\mathbb{C}}$.
3. Let k be a number field and S a finite set of valuations of k containing the archimedean valuations. The third is to determine when there exists a Zariski-dense set R of S-integral points on the complement of a union of divisors in projective space $\mathbb{P}^n_{\overline{k}}$, defined over k, where k is a number field and S is a finite set of valuations of k containing the archimedean valuations.
Here is a work joining with Nguyen Huu Kien.

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Gene Freudenburg (Western Michigan U.)
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Title : Canonical Factorization of the Quotient Morphism for an Affine $\mathbb{G}_a$-Variety.

Abstract : Working over a ground field k of characteristic 0, this talk introduces an algorithm to calculate the degree modules of a locally nilpotent derivation D of a commutative k-domain in the case where $\ker D$ is noetherian. Using this, we study the quotient morphism $\pi : X \to Y$ for an affine $\mathbb{G}_a$-variety X with affine quotient Y. We show that the associated degree modules give a uniquely determined sequence of dominant $\mathbb{G}_a$-equivariant morphisms, $X = X_r \to X_{r-1} \to \cdots \to X_1 \to X_0 = Y$, where X_i is an affine variety and $X_{i+1} \to X_i$ is birational for each $i \geq 1$. This is the *canonical factorization* of π. The algorithm is applied to compute the canonical factorization for the homogeneous $(2,5)$-action on $\mathbb{A}^3$. By a fundamental result of Kaliman and Zaidenberg,

any birational morphism is an affine modification, and for this example, each mapping in the canonical factorization is presented as a $\mathbb{G}_a$-equivariant affine modification. In another application, we construct a triangular R-derivation Δ of $R[X, Y, Z]$ with a slice, where $R = \mathbb{C}^{[2]}$. The kernel of Δ is a $\mathbb{C}^2$-fibration over R, but it is not known if this fibration is trivial. Also, we formulate the Freeness Conjecture, which asserts that $\mathbb{C}^{[3]}$ is a free module over $\ker D$ for any locally nilpotent derivation D of $\mathbb{C}^{[3]}$.

Rajendra Vasant Gurjar (Tata Institute, Mumbai)
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Title : The quotient of a smooth integral homology 3-fold modulo the action of $(\mathbb{C}, +)$ is smooth

Abstract : *Let X be a smooth affine 3-fold with trivial integral homology. Then for any non-trivial regular action of the additive group $\mathbb{G}_a := (\mathbb{C}, +)$ the quotient $X//\mathbb{G}_a$ is smooth.*
For general X as above the proof implicitly uses a difficult result of M. Miyanishi-S. Tsunoda about smooth surfaces with log Kodaira dimension $-\infty$. If $X = \mathbb{C}^3$ then we can avoid the use of this difficult result and get an accessible proof of Miyanishi's old result that $\mathbb{C}^3//\mathbb{G}_a \cong \mathbb{C}^2$.

Takashi Kishimoto (Saitama U.)
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Title : Families of affine ruled surfaces : Existence of cylinders

Abstract : The result due to Kaliman and Zaidenberg asserts that an $\mathbb{A}^2$-fibration (of any dimension) $f : X \to Y$ contains an $\mathbb{A}^2$-cylinder, i.e., there exists an open subset $U \subseteq Y$ such that $f^{-1}(U) \cong U \times \mathbb{A}^2$ and the restriction $f|_{f^{-1}(U)}$ is nothing but the projection onto the first factor. Instead of an $\mathbb{A}^2$-fibration itself, we consider more generally an affine morphism $g : X \to Y$ (of any dimension) whose general fibers are smooth affine surfaces equipped with an $\mathbb{A}^1$-fibration. Different from the case of $\mathbb{A}^2$-fibrations just mentioned, in general such a g can not be factored by means of an $\mathbb{A}^1$-fibration even if we shrink the base Y however. Nevertheless, we can show that there exist an open affine subset $Y^* \subseteq Y$ and a finite étale morphism $\widetilde{Y} \to Y^*$ such that the corresponding fiber product $X \times_{Y^*} \widetilde{Y} \to \widetilde{Y}$ is decomposed by a suitable $\mathbb{A}^1$-fibration. This is a joint work with Adrien Dubouloz.

Mariusz Koras (Warsaw U.)
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Title : Smooth embeddings of $\mathbb{C}^*$ into the plane

Abstract : I present the final classification of curves isomorphic to $\mathbb{C}^*$ contained in $\mathbb{C}^2$, up to authomorphism of $\mathbb{C}^2$. The list confirms the classification obtained by Borodzik and Zoladek under certain unproved conjecture. This is a joint work with Pierrette Cassou- Nogues, Karol Palka and Peter Russell.

Hanspeter Kraft (U. Basel)
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Title : Automorphism Groups of Affine Varieties

Abstract : In 1966 Shafarevich introduced the notion of “infinite dimensional algebraic group”, shortly “ind-group”. His main application was the automorphism group of affine n-space A^n for which he claimed some interesting properties. Recently, jointly with J.-Ph. Furter we showed that the automorphism group of any finitely generated (general) algebra has a natural structure of an ind-group, and we further developed the theory.
It turned out that some properties well-known for algebraic groups carry over to ind-groups, but others do not. E.g. every ind-group has a Lie algebra, but the relation between the group and its Lie algebra still remains unclear. As another by-product of this theory we get new interpretations and a better understanding of some classical results, together with short and nice proofs.
An interesting “test case” is $Aut(A^2)$, the automorphism group of affine 2-space, because this group is the amalgamated product of two closed subgroups which implies a number of remarkable properties. E.g. a conjugacy class of an element $g \in Aut(A^2)$ is closed if and only if g is semi-simple, a result well-known for algebraic groups. A generalisation of this to higher dimensions would have very strong and deep consequences, e.g. for the so-called linearisation problem.
One of the highlights is the following result. If X is a connected variety whose automorphism group $Aut(X)$ is isomorphic to $Aut(A^n)$ as an ind-group, then X is isomorphic to A^n as a variety. This is one special answer to the general question what kind of information about an affine variety X can be retrieved from its automorphism group $Aut(X)$.

Peter Kuchment (Texas A&M U.)
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Title : Analytic properties of dispersion relations and spectra of periodic operators

Abstract : The talk will survey some known results and unresolved problems concerning analytic properties of dispersion relations and their role in various spectral theory problems for periodic operators of mathematical physics, such as spectral structure, embedded impurity eigenvalues, Greens function asymptotics, Liouville theorems, etc.

Frank Kutzschebauch (U. Bern)
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Title : Big automorphism groups in Affine Algebraic Geometry and Complex Analysis

Abstract : We give some ideas about the influence of Mikhail Zaidenberg’s second most cited paper : "Affine modifications and affine hypersurfaces with a very transitive automorphism group" (with Sh. Kaliman).

Kevin Langlois (Univ. of Düsseldorf)
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Title : Motivic integration and actions of algebraic groups

Abstract : Since its creation by Maxim Kontsevich in 1995, motivic integration aims to make a connection between harmonic analysis over a local field and algebraic geometry over a field k of characteristic zero. It appears as an integration theory over $k((t))$ where the values of the integrals are of geometric nature, i.e., they correspond to motives or vitual classes of algebraic varieties.
In this talk, we will first give a gentle introduction to motivic integration and survey some classical results on stringy invariants. This invariant associates to any log-terminal variety X a certain motivic integral $\mathcal{E}(X)$. The volume $\mathcal{E}(X)$ coincides with the vitual class of X in the smooth case but turns out to be different in general.
In the last part of the talk, we will present a joint work with Clélia Pech and Michel Raibaut where we compute the stringy invariants for a specific class of log-terminal varieties admitting an action of a connected reductive group G. The G-action has the property that the general orbits are torus bundles over flag varieties and depend morphically on a smooth algebraic curve.

Mikhail Lyubich (IMS at Stony Brook)
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Title : On the dynamics of dissipative complex Henon maps

Abstract : Dissipative Henon maps are polynomial automorphism of $\mathbb{C}^2$ that can be viewed as perturbations of one-dimensional quadratic polynomials. We will discuss several themes in this area : classification of periodic Fatou components, problem of existence of wandering components, and stability & bifurcations in holomorphic families of Henon maps.

Masayoshi Miyanishi (Sanda U., Japan)
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Title : Affine threefolds with $\mathbb{G}_a$ actions

Abstract : Under the above title, we discuss the following two problems.
(1) Let $f : X \to Y$ be an $\mathbb{A}^1$-fibration from a smooth affine threefold to a normal affine surface. Is it true that for any closed point P of X there exists a smooth curve C passing through P and isomorphic to $\mathbb{A}^1$?
(2) Let f be obtained as the quotient morphism of X by a $\mathbb{G}_a$ action. Under a suitable condition like factoriality of X, describe the singularity of Y.
Concerning the problem (1), let F be an irreducible fiber component of f passing through P. If $\dim F = 1$ then F is isomorphic to $\mathbb{A}^1$, but we do not know the answer if $\dim F = 2$. In this case, it is likely that F is affine-ruled. But the normality of F is further necessary to show. In considering this problem, it is important to consider a proper morphism $p : V \to Y$ which is a $\mathbb{P}^1$-fibration and an irreducible fiber component G of p. If $\dim G = 1$, we can say that G is isomorphic to $\mathbb{P}^1$, but we do not the case $\dim G = 2$.

Concerning the problem (2), the most important singularity that may occur on Y is E_8-singularity $x^2 + y^3 + z^5 = 0$. We discuss various cases for which this singularity really occurs.
This is a joint work with R.V. Gurjar, M. Koras, K. Masuda and P. Russell.

Fedor Pakovich (Ben Gurion U., Beer-Sheva)
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Title : On semiconjugate rational functions

Abstract : Let A, B be two rational functions of degree at least two on the Riemann sphere. The function B is said to be semiconjugate to the function A if there exists a non-constant rational function X such that the equality

$$A \circ X = X \circ B \tag{1}$$

holds. The semiconjugacy relation plays an important role in the classical theory of complex dynamical systems as well as in the new emerging field of arithmetic dynamics. In the talk we present a description of solutions of (1) in terms of two-dimensional orbifolds of non-negative Euler characteristic on the Riemann sphere. As an application, we provide new results about the classical problem of description of commuting rational functions to which equation (1) reduces for $A = B$.

Karol Palka (Inst. Math. PAN, Warsaw)
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Title : On the equivariant Generalized Jacobian Conjecture

Abstract : A smooth complex variety satisfies the G-equivariant Generalized Jacobian Conjecture if all its G-equivariant étale endomorphisms are proper. The conjecture generalizes the well-known Jacobian Conjecture, which is (despite many proofs available online) open already in dimension 2. It is therefore interesting to study it for $\mathbb{Q}$-acyclic varieties, especially the ones of negative logarithmic Kodaira dimension, a natural class containing affine spaces. We will present new results (with A. Dubouloz) in dimension two, including a complete discussion for infinite groups G.

Pierre-Marie Poloni (U. Bern)
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Title : Counterexamples to the Complement Problem

Abstract : The complement problem is one of the "challenging problems in affine n-space" posed by Hanspeter Kraft in his Bourbaki seminar (1995). It asks the following : Suppose that two irreducible (algebraic) hypersurfaces of the complex affine n-space have isomorphic complements. Does this imply that they are isomorphic ?
In this talk, we will settle this question in every dimension n at least three by giving explicit counterexamples. Since we can arrange that one of the hypersurfaces is singular whereas the other is smooth, we have counterexamples in the analytic setting too.

Yury Prokhorov (Steklov Math. Inst., Moscow)
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Title : On compactifications of $\mathbb{C}^n$

Abstract : I will review some results related to the Hirzebruch problem on compactifications of the affine space. A new series of four-dimensional compactifications will be constructed. (This is a jointworkwith M. Zaidenberg.)

Bernard Shiffman (Johns Hopkins U., Baltimore)
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Title : Existence of flat holomorphic sections on compact Kähler manifolds.

Abstract : We consider sections in the spaces $SH^0(M, L^k)$ of L^2-normalized sections of the tensor powers L^k of a positive line bundle on a compact Kähler manifold M. In joint work with Zelditch in 2003, we showed that “most" of these sections have high peaks ; i.e., their sup-norms are unbounded and grow at the rate $\sqrt{\log k}$. A natural question is whether there exist uniformly bounded (or “flat") sequences of sections $s_k \in SH^0(M, L^k)$. For the case $M = CP^n$, Bourgain constructed uniformly bounded orthonormal bases for $H^0(CP^1, \mathcal{O}(k))$ in 1985 and recently for $H^0(CP^2, \mathcal{O}(k))$ (*AJM* 2016). We construct flat sequences for a positive line bundle on any compact Kähler manifold. In fact, we obtain sequences of orthonormal sets (with cardinality of the order $h^0(M, L^k)$) of sections in $H^0(M, L^k)$ that are uniformly bounded independent of k. (An improvement on this result was given in 2015 by Marzo and Ortega-Cerdà.) Our methods make use of the asymptotics of the Bergman kernel.

Muhammed Uludag (Galatasaray U., Istambul)
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Title : Jimm, a fundamental involution

Abstract : Dyer's outer automorphism of PGL(2,Z) induces an involution of the real line, which behaves very much like a kind of modular function. It has some striking properties : it preserves the set of quadratic irrationals sending them to each other in a non-trivial way and commutes with the Galois action on this set. It restricts to a highly non-trivial involution of the set of units of norm $+1$ of quadratic number fields. It conjugates the Gauss continued fraction map to the so-called Fibonacci map. It preserves harmonic pairs of numbers inducing a duality of Beatty partitions of N. It induces a subtle symmetry of Lebesgue's measure on the unit interval.
On the other hand, it has jump discontinuities at rationals though its derivative exists almost everywhere and vanishes almost everywhere. In the talk, I plan to show how this involution arises from a special automorphism of the infinite trivalent tree.

Yuri Zarhin (Penn State U., USA)
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Title : Jordan groups and birational automorphisms of conic bundles

Abstract : A classical theorem of Jordan asserts that each finite subgroup of the complex general linear group $\mathrm{GL}(n)$ is “almost commutative" : it contains a commutative normal subgroup, whose index is bounded by an universal constant that depends only on n. We discuss an analogue of this property for the groups of birational (and biregular) automorphisms of complex algebraic varieties, paying a special attention to the case of conic bundles over abelian varieties.
This is a report on a joint work with Tatiana Bandman.